

Attendance Problems. Solve for x.

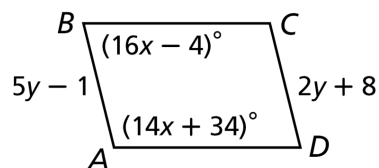
1. $16x - 3 = 12x + 13$

2. $2x - 4 = 90$

***ABCD* is a parallelogram. Find each measure.**

3. CD

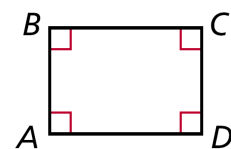
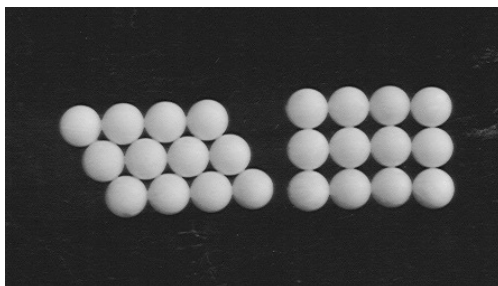
4. $m\angle C$



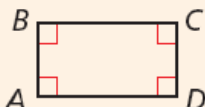
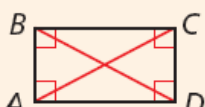
- I can prove and apply properties of rectangles, rhombuses, and squares.
- I can use properties of rectangles, rhombuses, and squares to solve problems.

Common Core: CC.9-12.G.CO.11 Prove theorems about parallelograms.

A second type of special quadrilateral is a *rectangle*. A **rectangle** is a quadrilateral with four right angles.

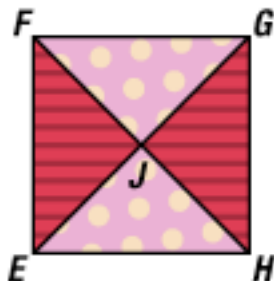
Rectangle $ABCD$ 

Theorems Properties of Rectangles

THEOREM	HYPOTHESIS	CONCLUSION
6-4-1 If a quadrilateral is a rectangle, then it is a parallelogram. (rect. \rightarrow \square)		$ABCD$ is a parallelogram.
6-4-2 If a parallelogram is a rectangle, then its diagonals are congruent. (rect. \rightarrow diags. \cong)		$\overline{AC} \cong \overline{BD}$

Refer to video example 1.

In this rectangular patch, the quilter cut the strips so that $EH = 12$ in. and $EG = 20$ in. Find FG and FJ .

**1 Craft Application**

An artist connects stained glass pieces with lead strips. In this rectangular window, the strips are cut so that $FG = 24$ in. and $FH = 34$ in. Find JG .

$$\overline{EG} \cong \overline{FH}$$

$$EG = FH = 34$$

$$JG = \frac{1}{2}EG$$

$$JG = \frac{1}{2}(34) = 17 \text{ in.}$$

Rect. \rightarrow diags. \cong

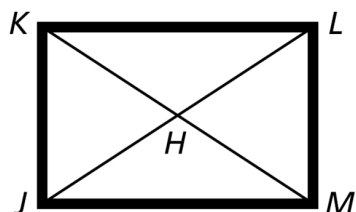
Def. of \cong segs.

$\square \rightarrow$ diags. bisect each other

Substitute and simplify.



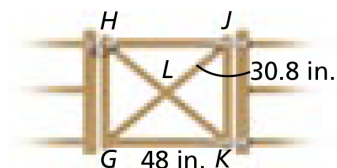
Example 1. A woodworker constructs a rectangular picture frame so that $JK = 50$ cm and $JL = 86$ cm. Find HM .



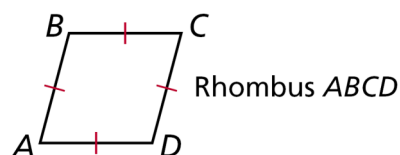
Guided Practice. The rectangular gate has diagonal braces. Find each length.

5. HJ

6. HK



A *rhombus* is another special quadrilateral. A **rhombus** is a quadrilateral with four congruent sides.



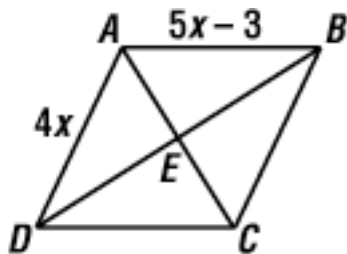
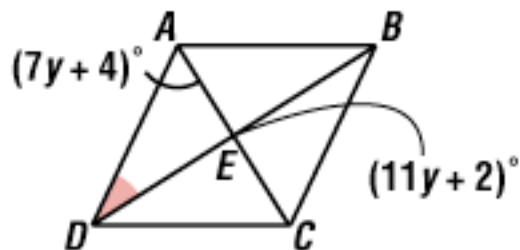
Theorems

Properties of Rhombuses

THEOREM	HYPOTHESIS	CONCLUSION
6-4-3 If a quadrilateral is a rhombus, then it is a parallelogram. (rhombus \rightarrow \square)		$ABCD$ is a parallelogram.
6-4-4 If a parallelogram is a rhombus, then its diagonals are perpendicular. (rhombus \rightarrow diags. \perp)		$\overline{AC} \perp \overline{BD}$
6-4-5 If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. (rhombus \rightarrow each diag. bisects opp. \angle s)		$\angle 1 \cong \angle 2$ $\angle 3 \cong \angle 4$ $\angle 5 \cong \angle 6$ $\angle 7 \cong \angle 8$

Refer to video example 2.

A. Find the length of CD.

B. Find $m\angle ADB$ **2****Using Properties of Rhombuses to Find Measures***RSTV* is a rhombus. Find each measure.**A** VT

$$ST = SR$$

Def. of rhombus

$$4x + 7 = 9x - 11$$

Substitute the given values.

$$18 = 5x$$

*Subtract 4x from both sides
and add 11 to both sides.*

$$3.6 = x$$

Divide both sides by 5.

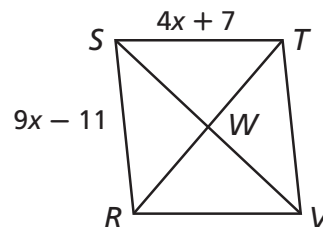
$$VT = ST$$

Def. of rhombus

$$VT = 4x + 7$$

Substitute 4x + 7 for ST.

$$VT = 4(3.6) + 7 = 21.4$$

Substitute 3.6 for x and simplify.*RSTV* is a rhombus. Find each measure.**B** $m\angle WSR$

$$m\angle SWT = 90^\circ$$

Rhombus \rightarrow diags. \perp

$$2y + 10 = 90$$

Substitute 2y + 10 for $m\angle SWT$.

$$y = 40$$

*Subtract 10 from both sides
and divide both sides by 2.*

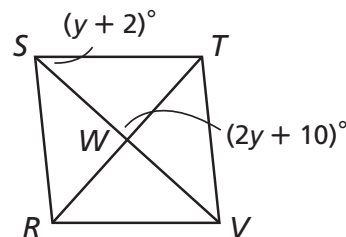
$$m\angle WSR = m\angle TSW$$

Rhombus \rightarrow each diag. bisects opp. \angle s

$$m\angle WSR = (y + 2)^\circ$$

Substitute y + 2 for $m\angle TSW$.

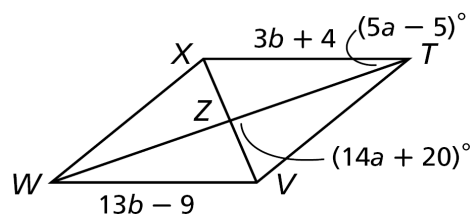
$$m\angle WSR = (40 + 2)^\circ = 42^\circ$$

Substitute 40 for y and simplify.

Example 2. TVWX is a rhombus. Find each measure.

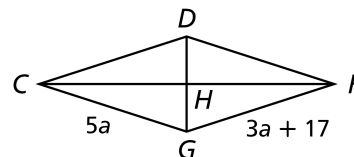
A. TV

B. $m\angle VTZ$



Guided Practice. CDFG is a rhombus.

7. Find the length of CD.



8. Find $m\angle GCH$ if $m\angle GCD = (b + 3)^\circ$ & $m\angle CDF = (6b - 40)^\circ$

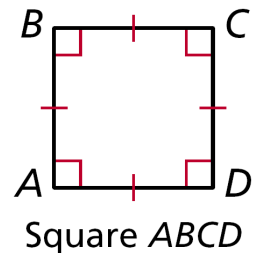
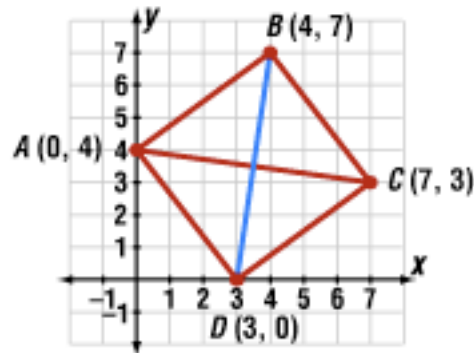
A **square** is a quadrilateral with four right angles and four congruent sides. In the exercises, you will show that a square is a parallelogram, a rectangle, and a rhombus. So a square has the properties of all three.

Helpful Hint

Rectangles, rhombuses, and squares are sometimes referred to as *special parallelograms*.

Refer to video example 3.

Show that the diagonals of square $ABCD$ are congruent perpendicular bisectors of each other.



3 Verifying Properties of Squares

Show that the diagonals of square $ABCD$ are congruent perpendicular bisectors of each other.

Step 1 Show that \overline{AC} and \overline{BD} are congruent.

$$AC = \sqrt{[2 - (-1)]^2 + (7 - 0)^2} = \sqrt{58}$$

$$BD = \sqrt{[4 - (-3)]^2 + (2 - 5)^2} = \sqrt{58}$$

Since $AC = BD$, $\overline{AC} \cong \overline{BD}$.

Step 2 Show that \overline{AC} and \overline{BD} are perpendicular.

$$\text{slope of } \overline{AC} = \frac{7 - 0}{2 - (-1)} = \frac{7}{3}$$

$$\text{slope of } \overline{BD} = \frac{2 - 5}{4 - (-3)} = \frac{-3}{7} = -\frac{3}{7}$$

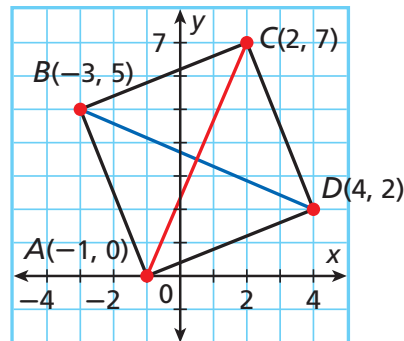
$$\text{Since } \left(\frac{7}{3}\right)\left(-\frac{3}{7}\right) = -1, \overline{AC} \perp \overline{BD}.$$

Step 3 Show that \overline{AC} and \overline{BD} bisect each other.

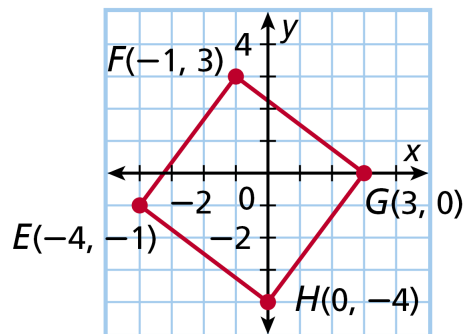
$$\text{mdpt. of } \overline{AC}: \left(\frac{-1 + 2}{2}, \frac{0 + 7}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$$

$$\text{mdpt. of } \overline{BD}: \left(\frac{-3 + 4}{2}, \frac{5 + 2}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$$

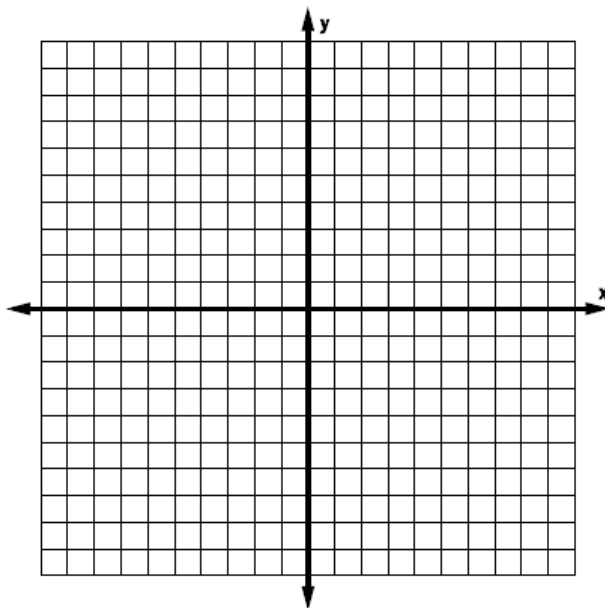
Since \overline{AC} and \overline{BD} have the same midpoint, they bisect each other. The diagonals are congruent perpendicular bisectors of each other.



Example 3. Show that the diagonals of square $EFGH$ are congruent perpendicular bisectors of each other.



9. Guided Practice. The vertices of square $STVW$ are $S(-5, -4)$, $T(0, 2)$, $V(6, -3)$, and $W(1, -9)$. Show that the diagonals of square $STVW$ are congruent perpendicular bisectors of each other.



6-4 Properties of Special Parallelograms (*p 424*) 11, 13-16, 36, 41-43, 44.